Role of Multipoint Space Missions In Unraveling Magnetic-Field Line Curvature

L'Aquila School – 2026 Course

The heliospheric space plasma physics in the era of multipoint space missions

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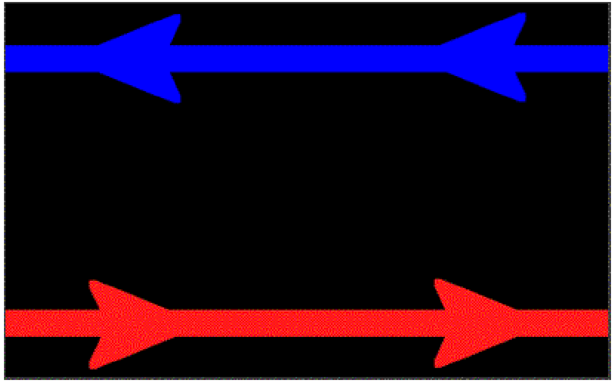
OUTLINE

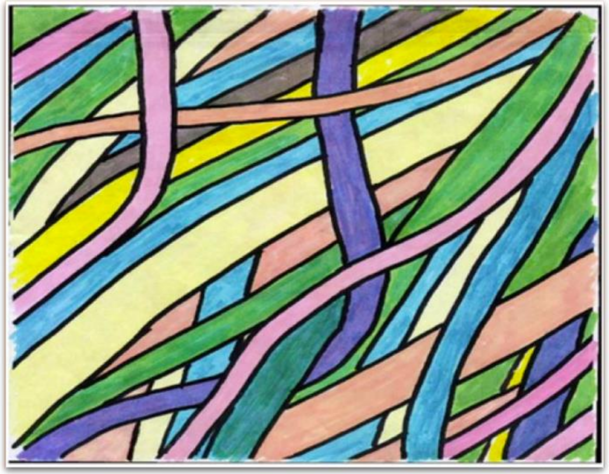
- Introduction
- Magnetic field line
  - curvature of field line
- Effects of curvature
- Simulation
  - Theory
  - Results
- MMS Observations
  - Results
- Effects of Anisotropy
- Summary

BACKGROUND

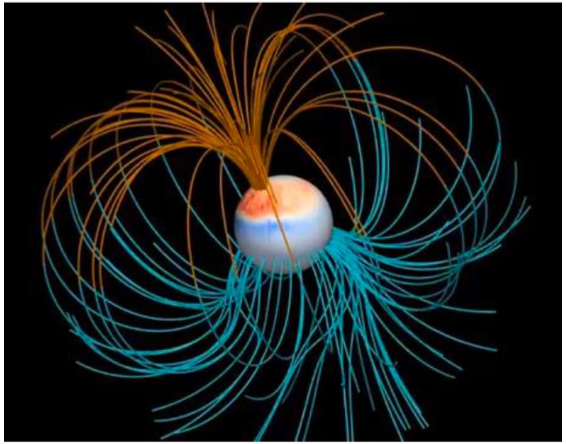
Magnetic field plays an essential role in many astrophysical and space plasmas



Magnetic reconnection
(credit: alifespentwondering.com)



Flux ropes
(Borovsky, 2008)

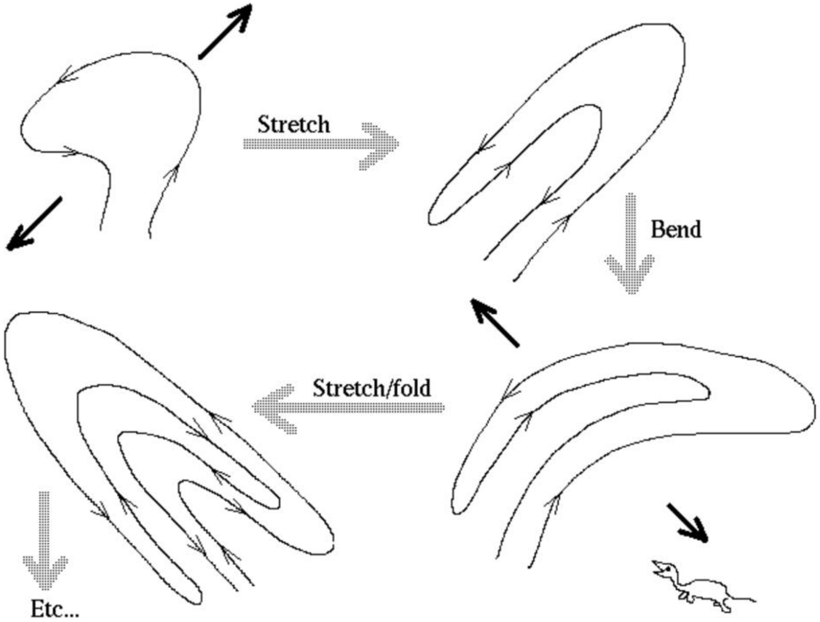


Dynamo
Sheyko+, 2016

BACKGROUND

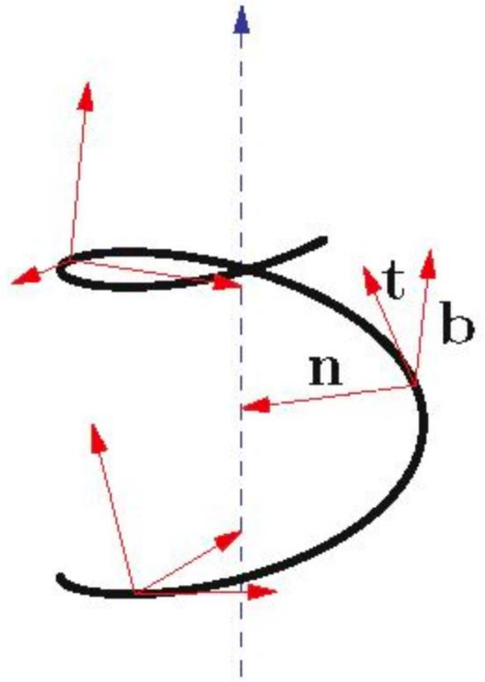
Magnetic field lines undergo stretch-twist-fold processes

Federrath+ 2011



Schekochihin+, 2002

MAGNETIC FIELD LINE



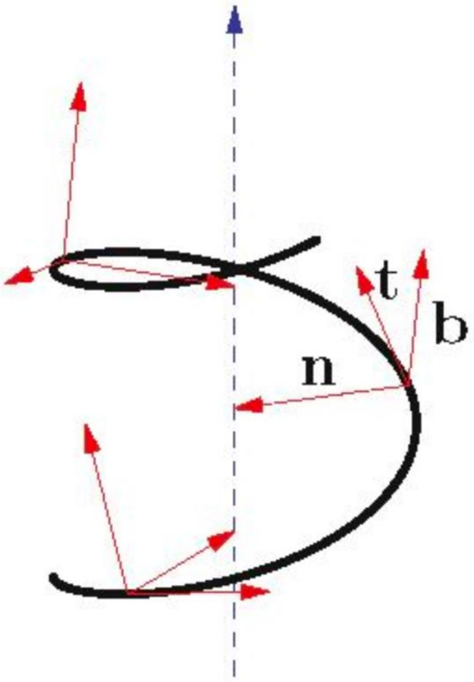
- Magnetic field line
  - Tangential vector $\boldsymbol{t} = \frac{\boldsymbol{B}}{|\boldsymbol{B}|} = \widehat{\boldsymbol{B}}$
  - Normal vector $\boldsymbol{n}$
  - Bi-normal vector $\boldsymbol{b}$

$$\frac{d\boldsymbol{t}}{ds} = \kappa\boldsymbol{n}$$

$$\frac{d\boldsymbol{n}}{ds} = -\kappa\boldsymbol{t} + \tau\boldsymbol{b}$$

$$\frac{d\boldsymbol{b}}{ds} = -\tau\boldsymbol{n}$$

CURVATURE OF MAGNETIC FIELD LINE



- Curvature

$$\kappa = \left|\frac{d\boldsymbol{t}}{ds}\right| = \left|\widehat{\boldsymbol{B}} \cdot \nabla\widehat{\boldsymbol{B}}\right|$$

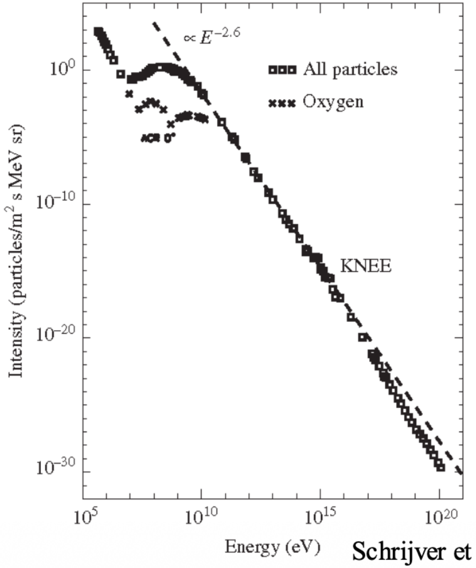
measuring how fast a curve is changing direction.

$$\boldsymbol{B} \cdot \nabla\boldsymbol{B} = B\frac{\partial B}{\partial s}\boldsymbol{t} + \kappa B^2\boldsymbol{n}$$

$$\kappa = \frac{|\widehat{\boldsymbol{B}} \times (\boldsymbol{B} \cdot \nabla\boldsymbol{B})|}{B^2} = \frac{f_n}{B^2}$$

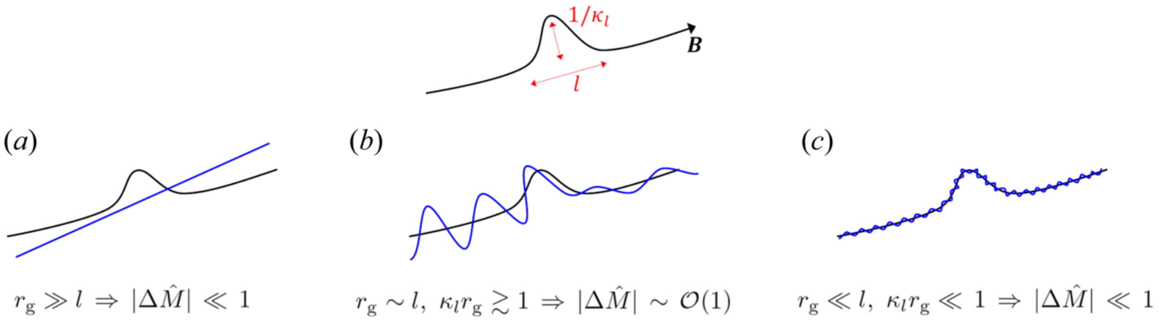
CURVATURE OF MAGNETIC FIELD LINE

- Particle Acceleration
  (curvature drift acceleration)
  Dahlin et al, *PoP*, 2014; Guo et al, *PRL*, 2014; Li et al, *ApJ*, 2017



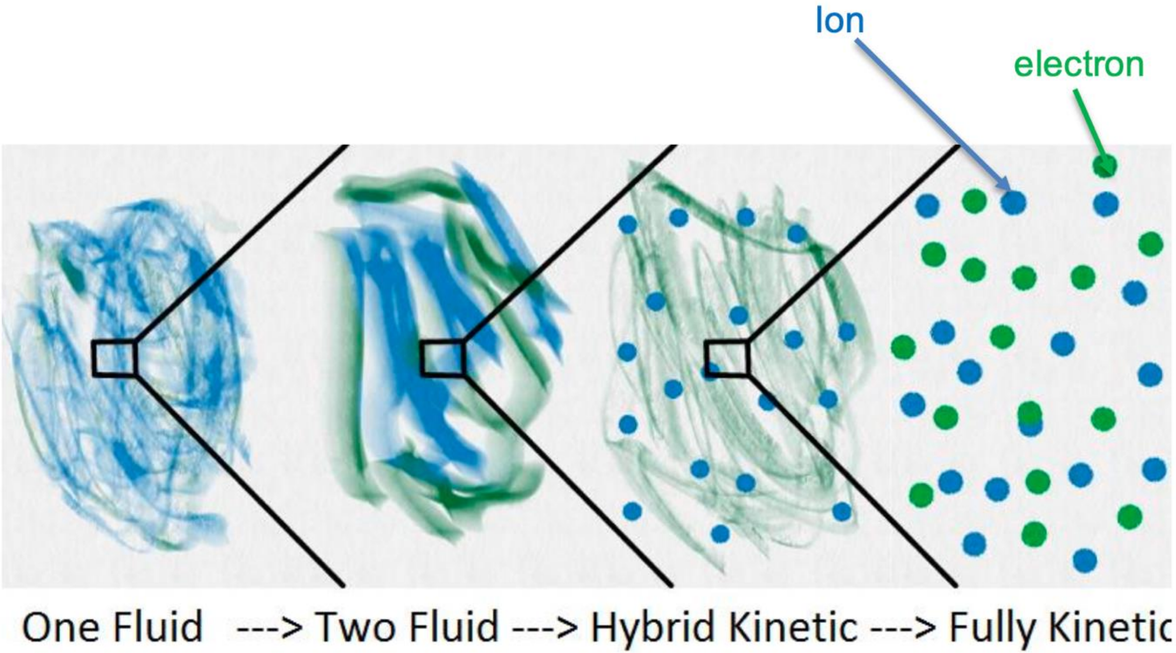
Schrijver et al, *Cambridge University Press*, 2010

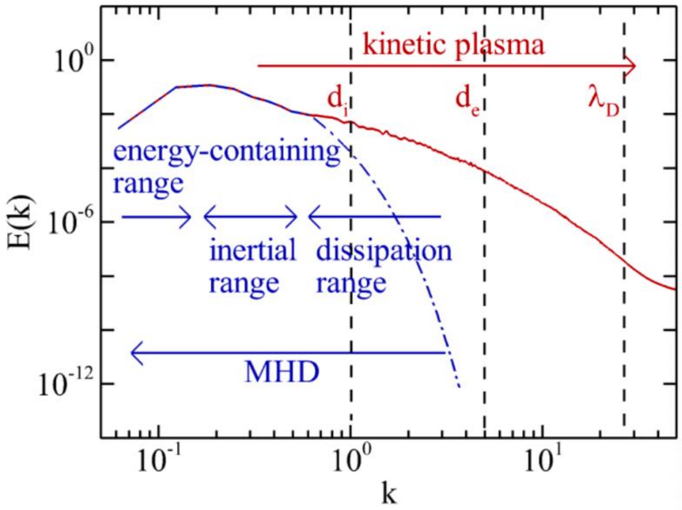
- Particle transport through localized interactions with sharp magnetic field bends



Lemoine+, 2023

NUMERICAL SIMULATION





Yang, *Springer*, 2019

SIMULATION SETUP

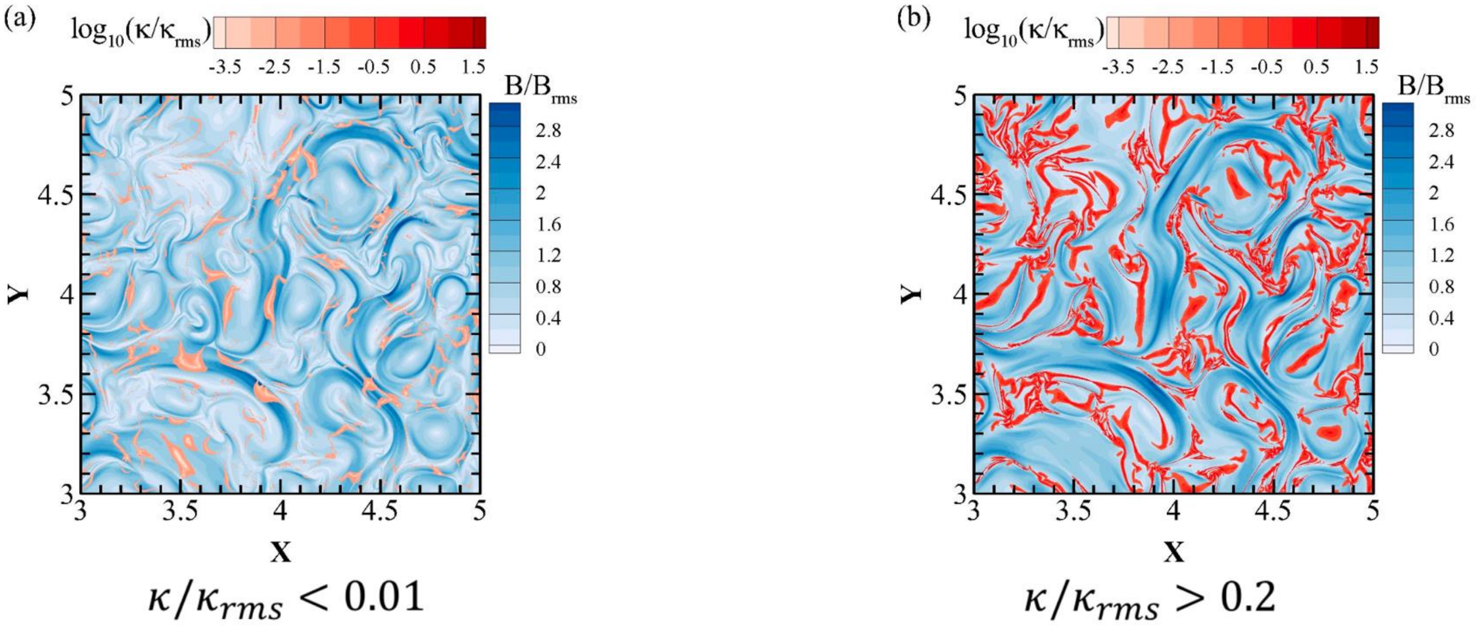
- 2D incompressible MHD simulation
  - Spectral method, L = 2π, N = 8192^2, Re = 5000

- 3D incompressible MHD simulation
  - Spectral method, L = 2π, N = 1024^3, Re = 2500, $B_0 = 0$

- 2.5D fully kinetic simulation
  - Particle-in-cell (PIC) method, L = 150 d_i, N = 4096^2, $B_0 = 1$, $m_i/m_e = 25$, ppg = 3200

Yang+ 2019

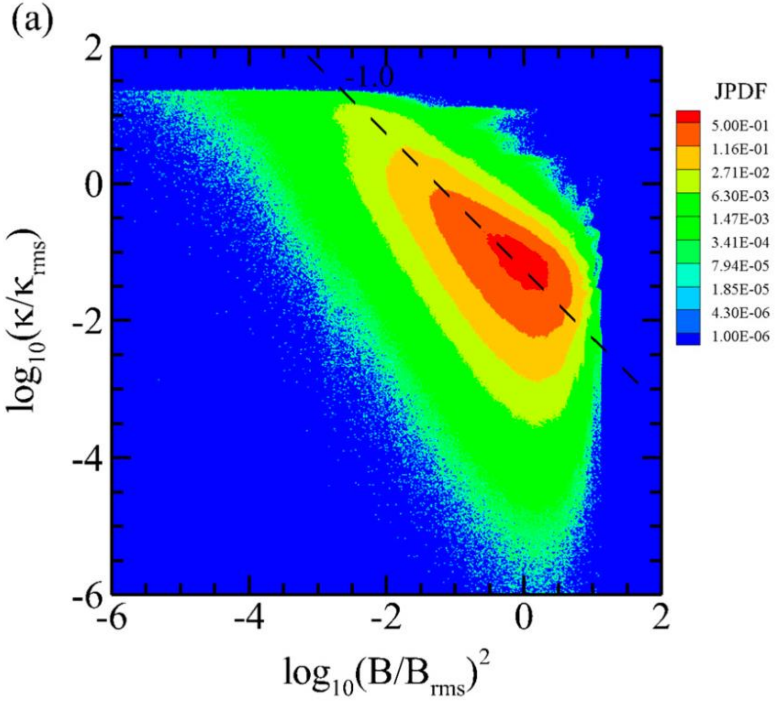
CURVATURE AND MAGNETIC FIELD



- Twisted lines (around the rims of magnetic islands) & isolated points (around magnetic island cores)
- Co-located with low magnetic magnitude

Yang+ 2019

CURVATURE AND MAGNETIC FIELD



$$\kappa = |\hat{\boldsymbol{B}} \cdot \nabla \hat{\boldsymbol{B}}| = \frac{|\hat{\boldsymbol{B}} \times (\boldsymbol{B} \cdot \nabla \boldsymbol{B})|}{B^2} = \frac{f_n}{B^2}$$

- Association between high κ and low B

2D:

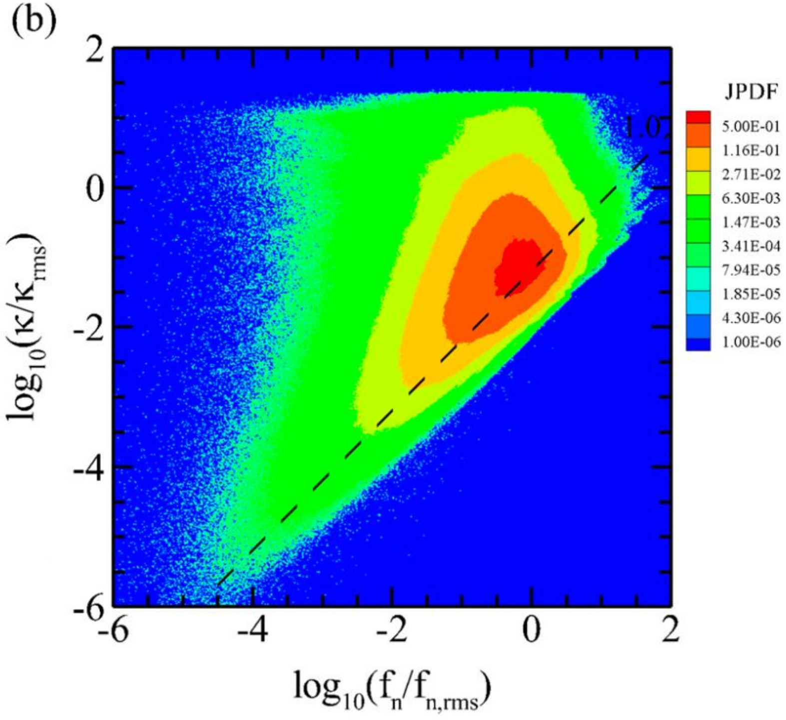
$$B_x, B_y \sim N(0, \sigma^2)$$
$$B^2 \sim \chi^2(2)$$
$$P_{\kappa\to\infty} \sim \kappa^{-2}$$

3D:

$$B_x, B_y, B_z \sim N(0, \sigma^2)$$
$$B^2 \sim \chi^2(3)$$
$$P_{\kappa\to\infty} \sim \kappa^{-2.5}$$

Yang+ 2019

CURVATURE AND MAGNETIC FIELD



$$\kappa = |\hat{\boldsymbol{B}} \cdot \nabla \hat{\boldsymbol{B}}| = \frac{|\hat{\boldsymbol{B}} \times (\boldsymbol{B} \cdot \nabla \boldsymbol{B})|}{B^2} = \frac{f_n}{B^2}$$

- Association between low κ and low f_n

2D:

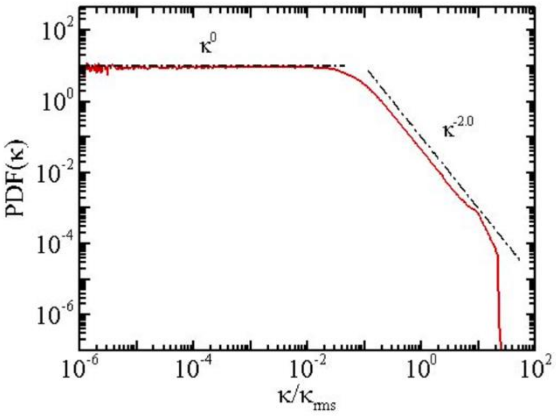
$$f_n \to 0 \sim N(0, \sigma^2)$$
$$P_{\kappa \to 0} \sim \kappa^0$$

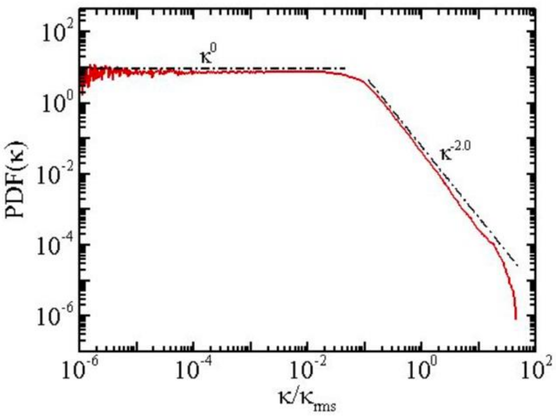
3D:

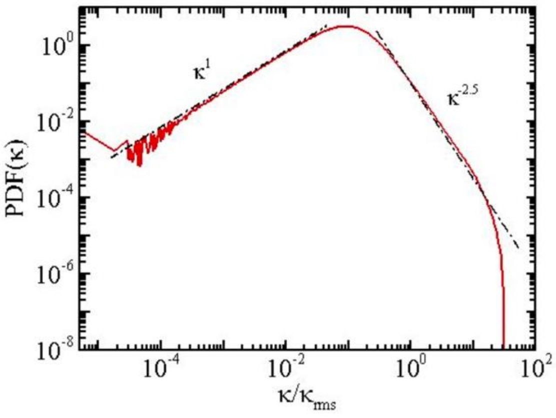
$$f_{n1}, f_{n2} \to 0 \sim N(0, \sigma^2)$$
$$f_n^2 \to 0 \sim \chi^2(2)$$
$$P_{\kappa \to 0} \sim \kappa^{+1}$$

Yang+ 2019

Curvature PDF







2D MHD
κ^0 low curvature tail
κ^{-2} high curvature tail

2.5D PIC
κ^0 low curvature tail
κ^{-2} high curvature tail

3D MHD
κ^1 low curvature tail
$\kappa^{-2.5}$ high curvature tail

Yang+ 2019

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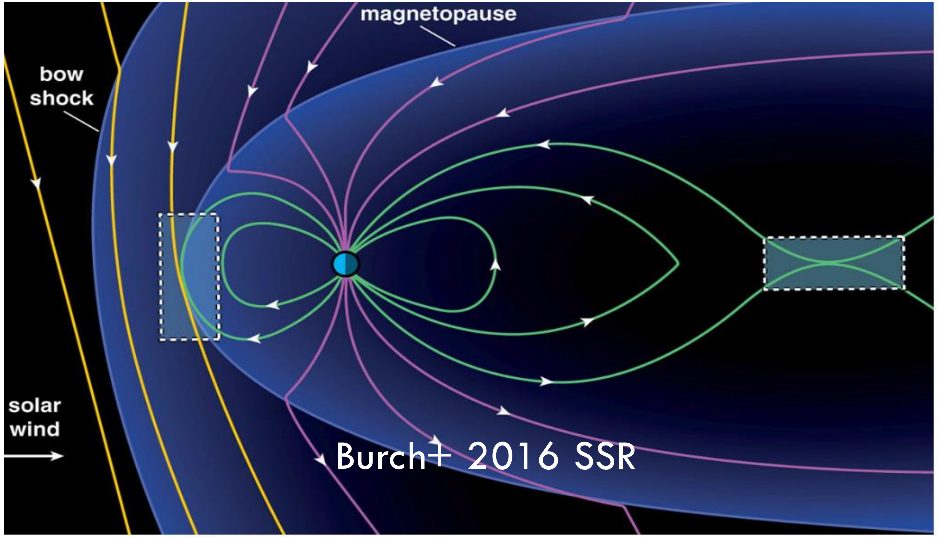
MANGETOSPHERIC MULTISCALE MISSION

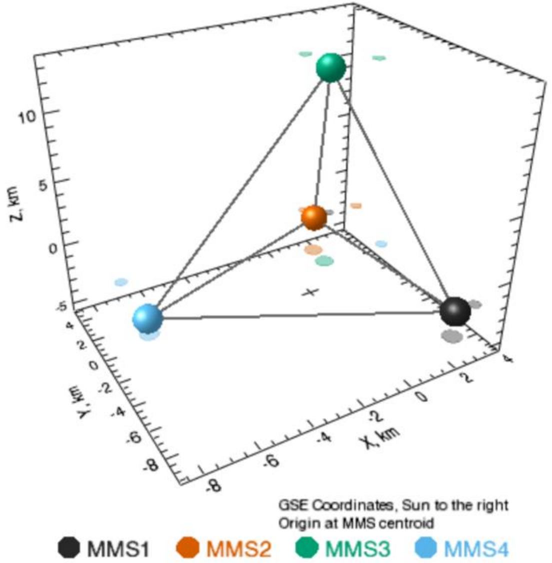
Source: MMS SDC

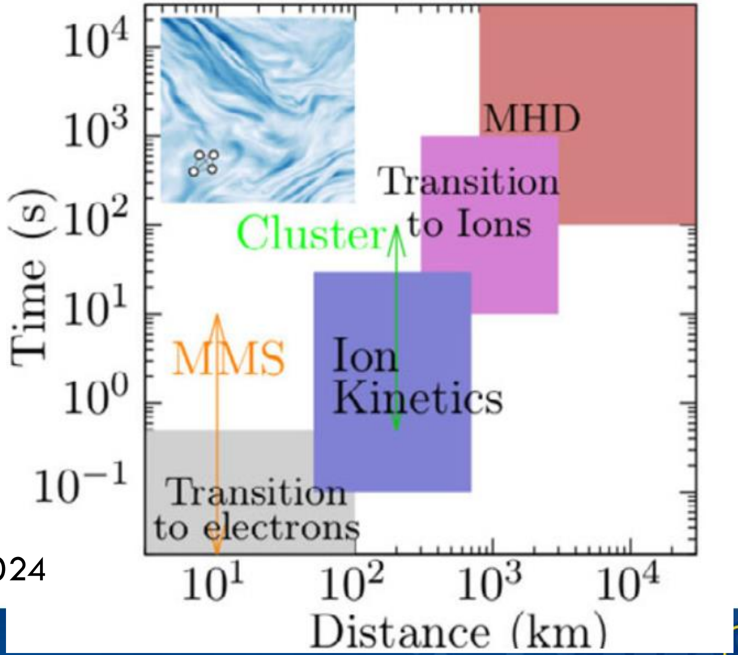


GRADIENT CALCULATION

4-spacecraft missions (e.g., Cluster and MMS) enable the study of spatial structures and calculation of gradient, such as curvature (Alex Chasapis talk)

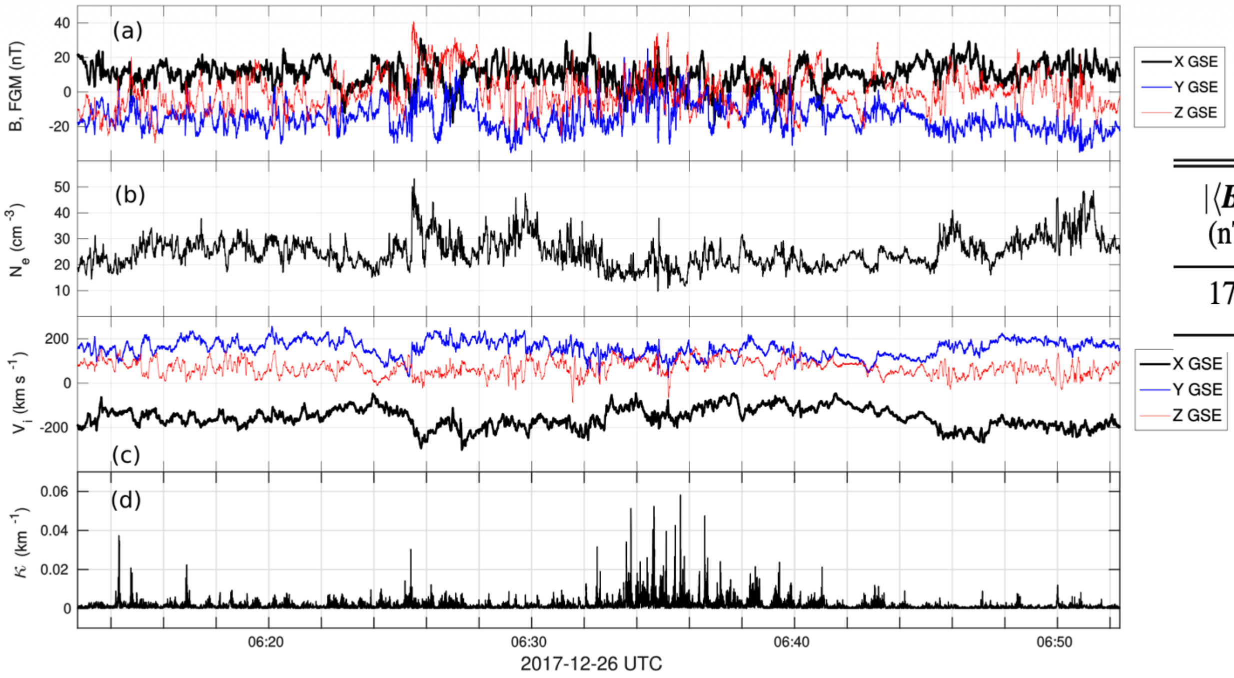
$$\kappa = \left|\widehat{\boldsymbol{B}} \cdot \nabla \widehat{\boldsymbol{B}}\right|$$





Klein+ 2024

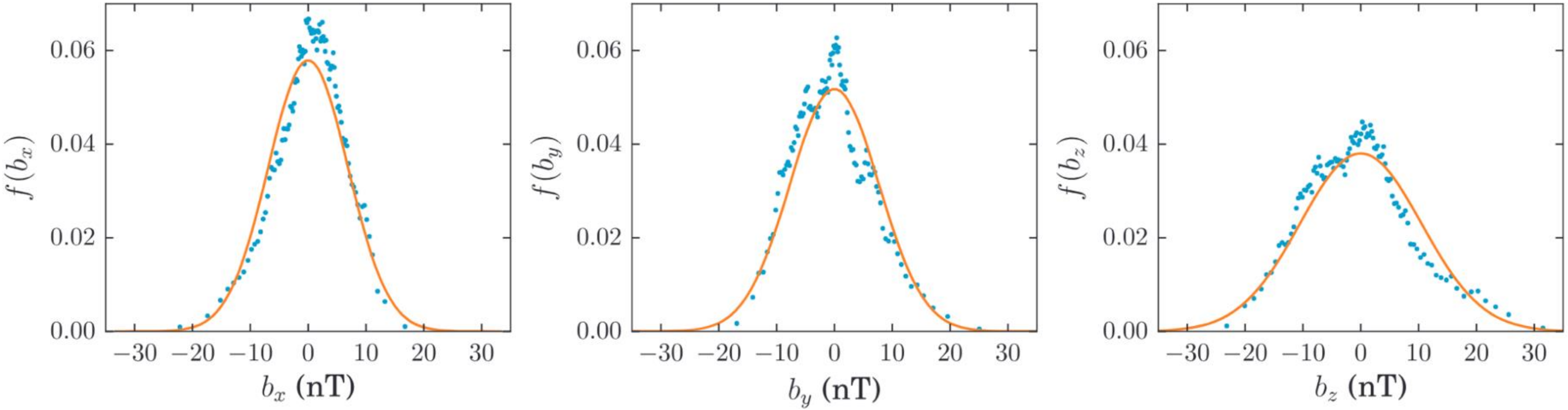
BURST-MODE DATA IN MAGNETOSHEATH



$\lvert\langle \boldsymbol{B} \rangle\rvert$ (nT)	$B_{rms}/\lvert\langle \boldsymbol{B} \rangle\rvert$	L (km)	d_i (km)	β_p
17.9	0.8	27	47	4.4

Bandyopadhyay+ 2020

GAUSSIANITY ASSUMPTION



Magnetic field components are (approximately) gaussian

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GAUSSIANITY ASSUMPTION

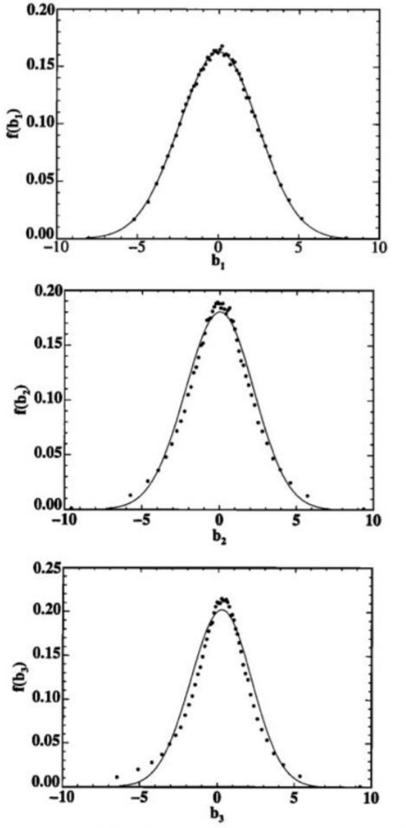


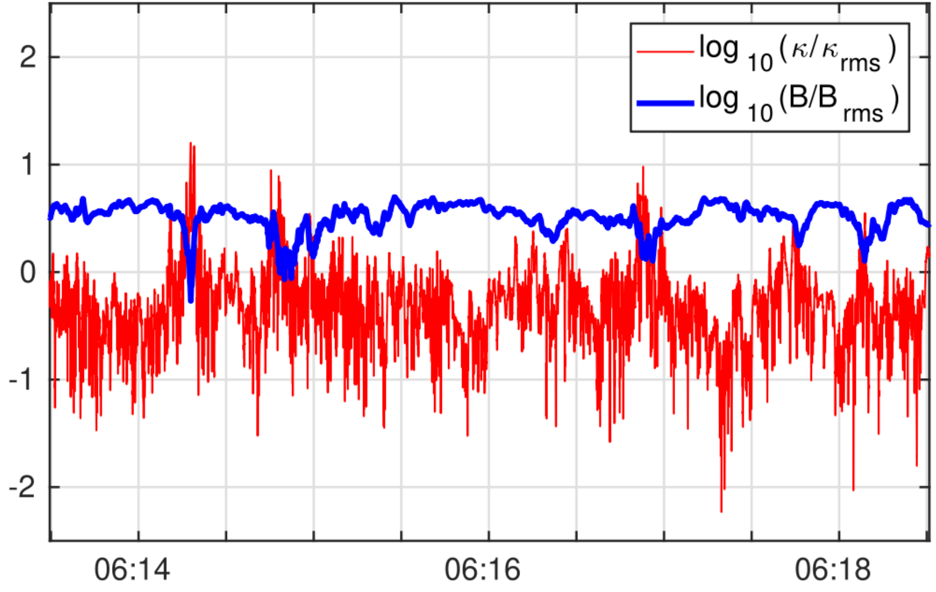
Table 1. Kurtoses and χ^2 Values

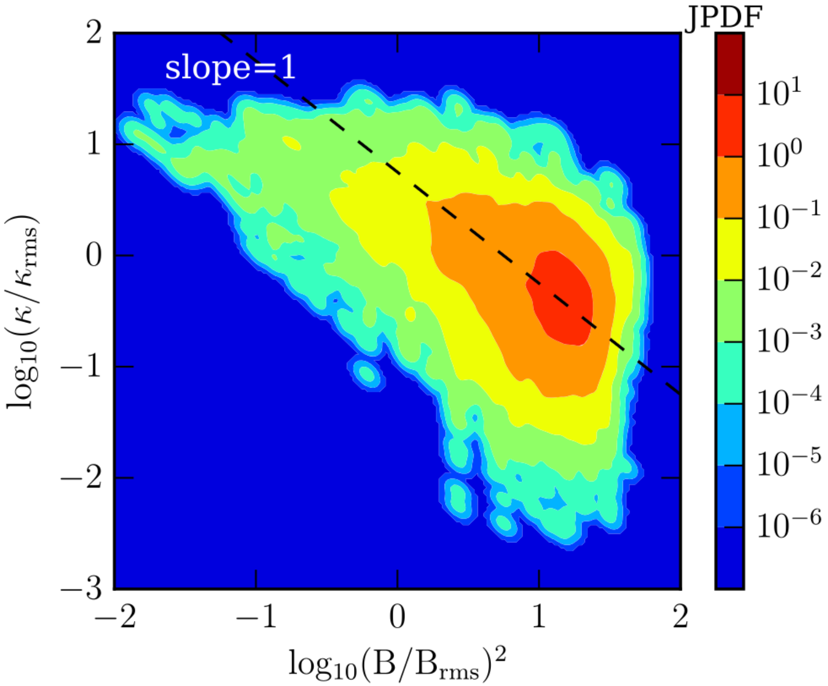
Data set	Parameter	Perpendicular Component 1	Perpendicular Component 2	Parallel Component
Omnitape 96h	Best fit χ^2	0.0003	0.0075	0.0168
	Kurtosis	2.99 ± 0.04	3.67 ± 0.05	4.17 ± 0.08
Ulysses 96h	Best fit χ^2	0.0017	0.0058	0.0345
	Kurtosis	2.95 ± 0.11	2.89 ± 0.16	4.10 ± 0.27
Ulysses 24h	Best fit χ^2	0.0050	0.0068	0.0727
	Kurtosis	3.18 ± 0.08	2.32 ± 0.05	5.98 ± 0.38

PDFs of Magnetic-field components are very well approximated by Gaussian in solar wind (fast and slow).

Padhye+ 2001

CURVATURE AND MAGNETIC FIELD





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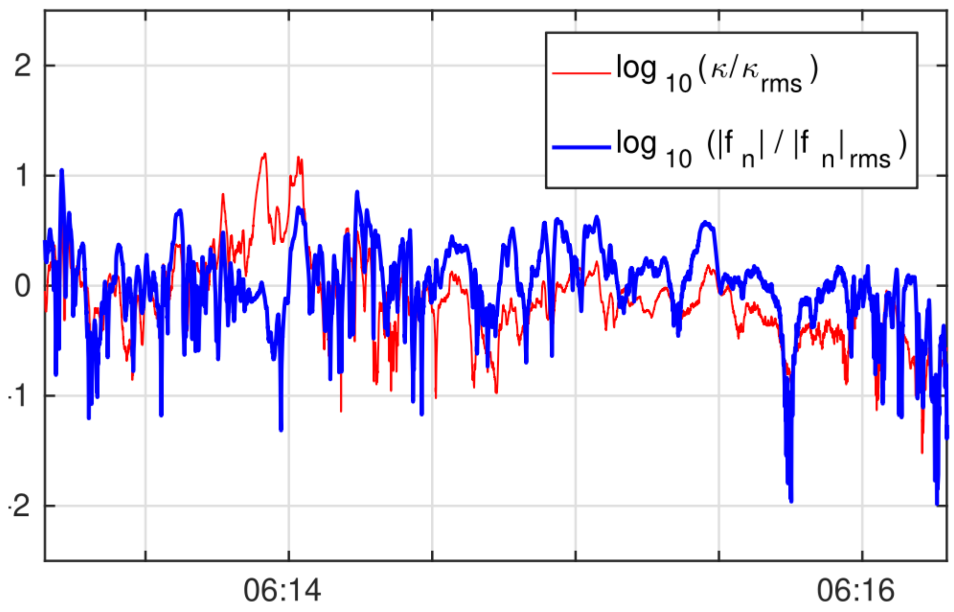
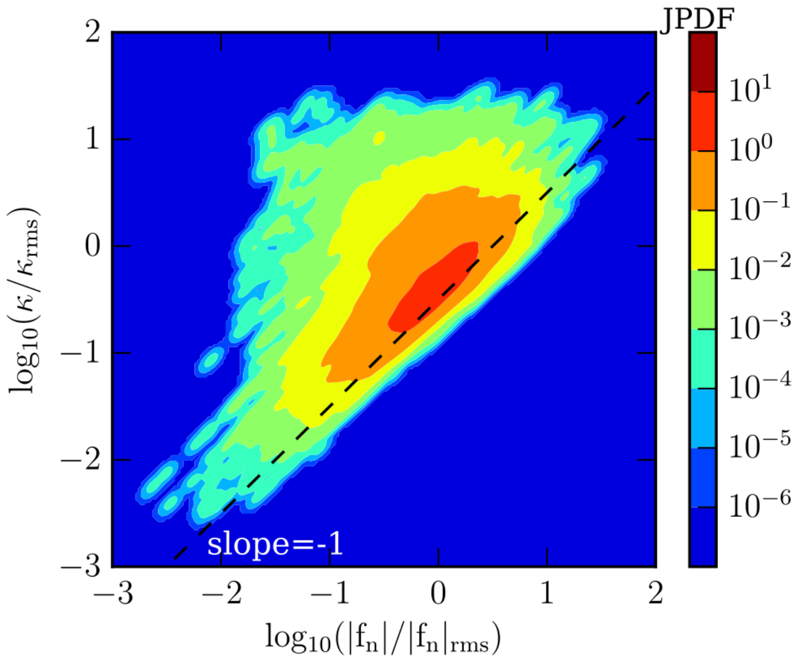
CURVATURE AND MAGNETIC FIELD





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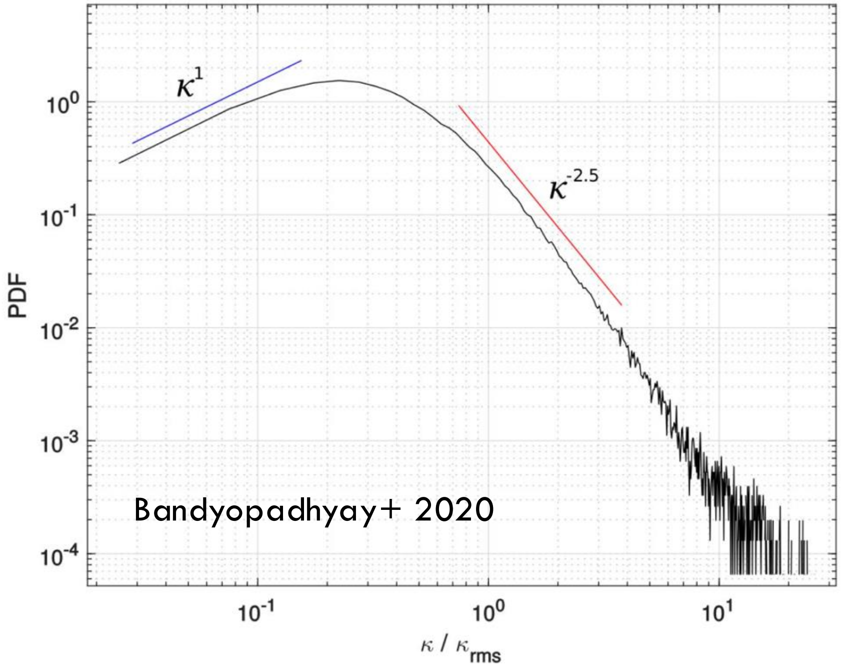
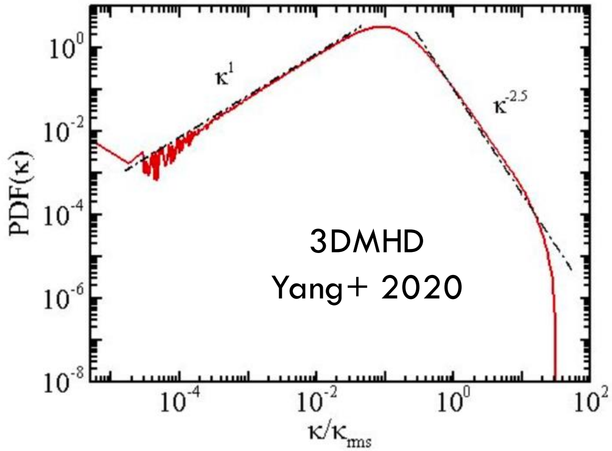
CURVATURE PDF





Consistent with theory

OTHER SAMPLES

Interval	Parameter	b_x	b_y	b_z	$B_{rms}/\|\langle \mathbf{B} \rangle\|$	Shock Type
26 December 2017, 06:12:43 - 06:52:23	χ^2	0.018	0.023	0.023	0.8	quasi-$\|$
	Kurtosis	3.56	3.50	3.23		
11 January 2016, 00:57:04 - 01:00:34	χ^2	0.031	0.020	0.067	1.5	quasi-$\perp$
	Kurtosis	3.76	3.06	4.5		
18 January 2017, 00:45:54 - 00:49:42	χ^2	0.020	0.067	0.113	1.8	quasi-$\|$
	Kurtosis	2.77	2.43	3.56		
27 January 2017, 08:02:03 - 08:08:03	χ^2	0.04	0.09	0.02	2.1	quasi-$\|$
	Kurtosis	3.14	3.08	3.29		
21 December 2017, 06:41:55 - 07:03:51	χ^2	0.085	0.022	0.019	2.1	quasi-$\|$
	Kurtosis	3.23	2.92	2.75		
21 December 2017, 07:21:54 - 07:48:01	χ^2	0.012	0.094	0.045	1.9	quasi-$\|$
	Kurtosis	4.03	2.83	2.61		
19 April 2018, 05:08:04 - 05:41:51	χ^2	0.011	0.014	0.011	3.1	quasi-$\perp$
	Kurtosis	3.41	3.81	3.46		
23 April 2018, 07:50:14 - 08:33:41	χ^2	0.019	0.035	0.027	1	quasi-$\perp$
	Kurtosis	3.54	3.47	3.73		
27 October 2018, 09:13:14 - 09:57:41	χ^2	0.017	0.010	0.029	2.5	quasi-$\|$
	Kurtosis	3.07	3.28	2.86		
21 November 2018, 16:10:14 - 16:55:31	χ^2	0.010	0.049	0.009	0.9	quasi-$\perp$
	Kurtosis	3.84	3.75	2.92		
29 November 2018, 22:42:34 - 23:31:01	χ^2	0.008	0.005	0.008	5	quasi-$\|$
	Kurtosis	3.04	2.90	3.10		
05 December 2018, 14:53:23 - 15:20:13	χ^2	0.019	0.015	0.013	7.5	quasi-$\|$
	Kurtosis	3.39	2.95	2.82		
11 January 2019, 03:22:23 - 03:52:23	χ^2	0.011	0.02	0.028	2.0	quasi-$\|$
	Kurtosis	3.52	2.56	2.94		
05 April 2019, 10:58:33 - 11:25:52	χ^2	0.024	0.035	0.086	1.9	quasi-$\|$
	Kurtosis	2.66	2.69	2.63		

- Similar results found for all other intervals.
- All intervals with large B_{rms}/B_0

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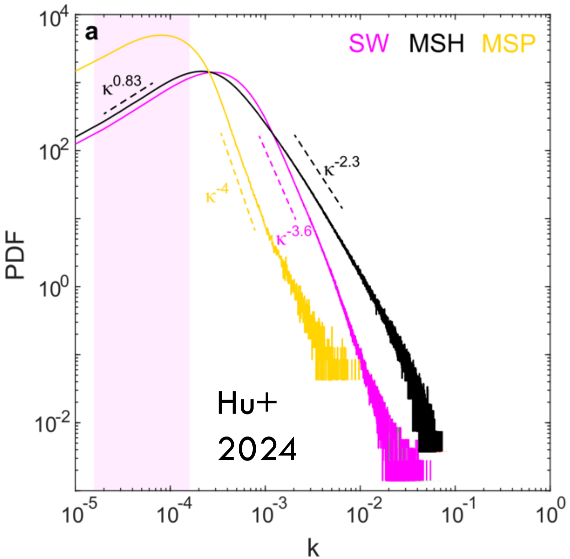
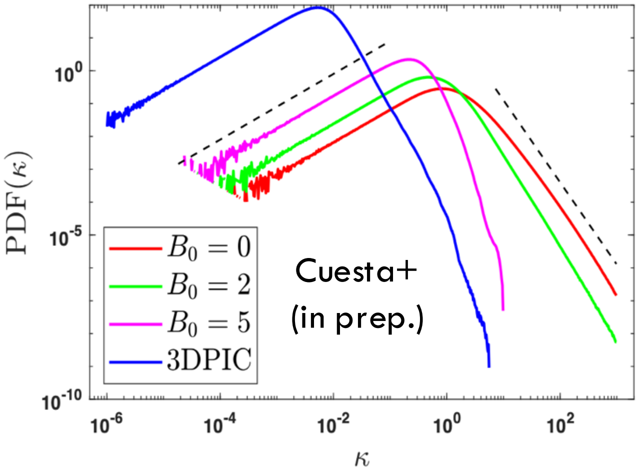
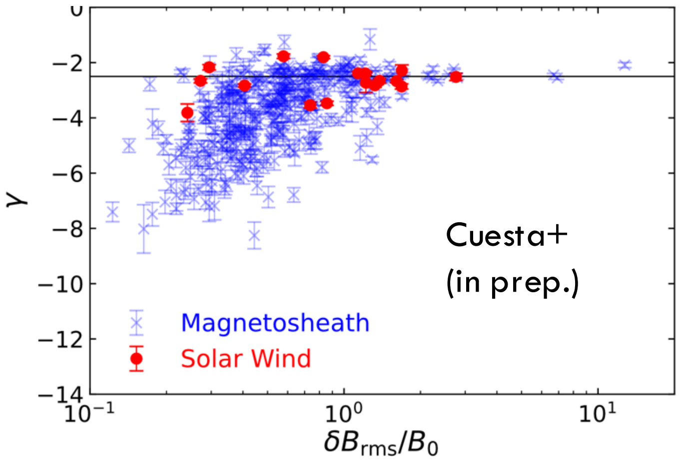
EFFECT OF THE MEAN MAGNETIC FIELD







- Low curvature: κ^{+1}
- High curvature: steeper than $\kappa^{-2.5}$
- Anisotropy effect?

SUMMARY

- Accurate measurement of curvature is possible with MMS data
- Different power laws are observed at low and high curvature values

Isotropic	2D MHD/2.5D PIC	3D MHD
$\kappa \rightarrow 0$	κ^{0}	κ^{1}
$\kappa \rightarrow \infty$	κ^{-2}	$\kappa^{-2.5}$
MMS	Solar Wind	Magnetosheath
$\kappa \rightarrow 0$	κ^{1}	κ^{1}
$\kappa \rightarrow \infty$	$\kappa^{\leq -2.5}$	$\kappa^{\leq -2.5}$

Thank you

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